Math 3450 - Homework # 1 - Sets
Part B - Proofs and Conceptual Questions

Part 1 - Conceptual

1. True or False: $\{1\} \in \mathcal{P}(\{1, 2\})$

2. True or False: $\{1\} \subseteq \mathcal{P}(\{1, 2\})$

3. Show that the following if-then statement is false by giving a counter-example: Let A, B, C be sets. If $A \cap B \neq \emptyset$ and $B \cap C \neq \emptyset$, then $A \cap C \neq \emptyset$.

Part 2 - Proofs

4. Prove that $\{12n \mid n \in \mathbb{Z}\} \subseteq \{2n \mid n \in \mathbb{Z}\} \cap \{3n \mid n \in \mathbb{Z}\}$.

5. Prove that $\{9^n \mid n \in \mathbb{Z}\} \subseteq \{3^n \mid n \in \mathbb{Z}\}$, but $\{9^n \mid n \in \mathbb{Z}\} \neq \{3^n \mid n \in \mathbb{Z}\}$.

6. Let $A = \{2k \mid k \in \mathbb{Z}\}$ and $B = \{3n \mid n \in \mathbb{Z}\}$. Prove that $A \cap B = \{6m \mid m \in \mathbb{Z}\}$.

7. Let A, B, C, D be sets.

   (a) Prove that if $A \subseteq B$, then $A \cup C \subseteq B \cup C$.

   (b) Prove that if $A \subseteq B$ then $A \subseteq B \cup C$.

   (c) Prove that if $A \subseteq B$, then $A - C \subseteq B - C$.

   (d) Prove that $A \subseteq B$ if and only if $A - B = \emptyset$.

   (e) Prove that $A \subseteq B$ if and only if $A \cap B = A$.

   (f) Prove that if $B \subseteq C$, then $A \times B \subseteq A \times C$.

   (g) Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

   (h) Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

   (i) Prove that $A \cap (B \cap C) = (A \cap B) \cap C$.

   (j) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

8. Let A and B be sets.

    (a) Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

    (b) Prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

    (c) Give an example where $\mathcal{P}(A) \cup \mathcal{P}(B) \neq \mathcal{P}(A \cup B)$.

9. Let A and B be sets. Prove that $A - B$ and B are disjoint.

10. Let A and B be sets. Suppose that $B \neq \emptyset$ and $A \times B \subseteq B \times C$. Prove that $A \subseteq C$.